

# Dynamical and scaling properties of $\nu = \frac{5}{2}$ interferometer

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We calculate the non-linear I-V tunneling curves for a two-point-contact tunneling junction between two edges of the  $\nu = \frac{5}{2}$  non-abelian fractional quantum Hall state. The non-linear I-V tunneling curves are calculated for both cases with and without an  $e/4$  non-abelian quasiparticle between the two contacts. We confirm that, within a dynamical edge theory, the presence of the  $e/4$  quasiparticle between the two contacts destroys the interference between the two tunneling paths. We also calculate how the interference reappears as the  $e/4$  quasiparticle is moved closer to an edge.

## INTRODUCTION

The possibility of having anyons, particles in two dimensional systems which obey fractional statistics in between bosons and fermions, is a remarkable prediction by theory [1, 2]. To observe anyons experimentally is hard however, as the signature of fractional statistics lies not in the observation of anyonic thermodynamics, but rather in the observation of Aharonov-Bohm type interference in quasiparticle motions.

For abelian anyons, the Aharonov-Bohm interference of a quasiparticle is described by a complex phase [3]. Such a phase can be changed in discrete steps when some extra quasiparticles are enclosed by the two interference paths.

However, for the more exotic non-abelian anyons, the Aharonov-Bohm interference is described by matrices that act on a global Hilbert space of topologically protected degenerate states [4, 5, 6], which represent a potentially decoherence free [7] qubit realization [8]. Such an unusual fractional statistics is called non-abelian fractional statistics. For non-abelian anyons not only the phase but also the magnitude of the interference can be affected. Complete destructive interference would e.g. be a manifestation of non-abelian statistics [9, 10, 11, 12].

A candidate system that allows non-abelian anyons is the Moore-Read Pfaffian description [5, 13, 14, 15] for the  $\nu = \frac{5}{2}$  fractional quantum Hall state in which interference would occur in the tunneling current induced by quantum point contacts. For the Moore-Read state, a setup with destructive interference has been predicted theoretically [9, 10, 11, 12, 16]. In such a setup, an  $e/4$  non-Abelian quasiparticle is trapped in a central island between two tunneling contacts. In the regime of weak tunneling the temperature and bias dependence of the tunneling conductance can be calculated exactly through the edge state theory [15].

The proposed interference in the  $\nu = \frac{5}{2}$  state is destructive to leading order in the point contact tunneling amplitudes. Higher order contributions can potentially restore some interference. Here, we consider one such higher order process. To be specific we allow for the exchange of the charge-neutral fermionic quasiparticle in

the  $\nu = 5/2$  Pfaffian state between the central island and one of the two edges. Such a process gives rise to a non-vanishing interference pattern. This contribution can be distinguished from leading order interference pattern through the different scaling dependences on temperature and bias.

There are other processes which could potentially restore interference, for example the tunneling of the island quasiparticle into one of the edges. Such a process is rather complicated to describe theoretically however. Through a detailed numerical calculation, it was found that the neutral excitations on the edge have much lower energy scale than the charged excitations.[17] This suggests the possibility that the neutral-fermion tunneling may dominate the  $e/4$ -quasiparticle tunneling.

The tunneling of the neutral fermionic quasiparticle also has the advantage that it can be treated in the existing framework of leading order perturbative calculation. Yet it touches on the fundamental properties of the non-abelian anyon state as the neutral fermion tunneling flips the occupation of the topologically protected zero mode.

## INTERFEROMETER FOR THE MOORE-READ STATE WITH DESTRUCTIVE INTERFERENCE

### Tunneling conductance in FQH interferometer, average and amplitude

The interferometer concept that is used for generic quantum Hall fluids is depicted in the inset of Fig. 1b. Two edges, one left- and one right-moving, of a quantum hall fluid are brought closer together by applying quantum point contacts; an applied bias voltage between the two edges will then induce a tunneling current of quasiparticles at these point contacts. In case of two or more point contacts, the multiple ways in which a quasiparticle can tunnel can cause interference of these paths; the presence of a non-trivial central island between the point contacts can affect the interference due to the fractional statistics of the quasiparticles.

The low-energy gapless modes are described by the appropriate chiral Luttinger liquid edge state theory [18]. The best candidate to describe the fractional quantum

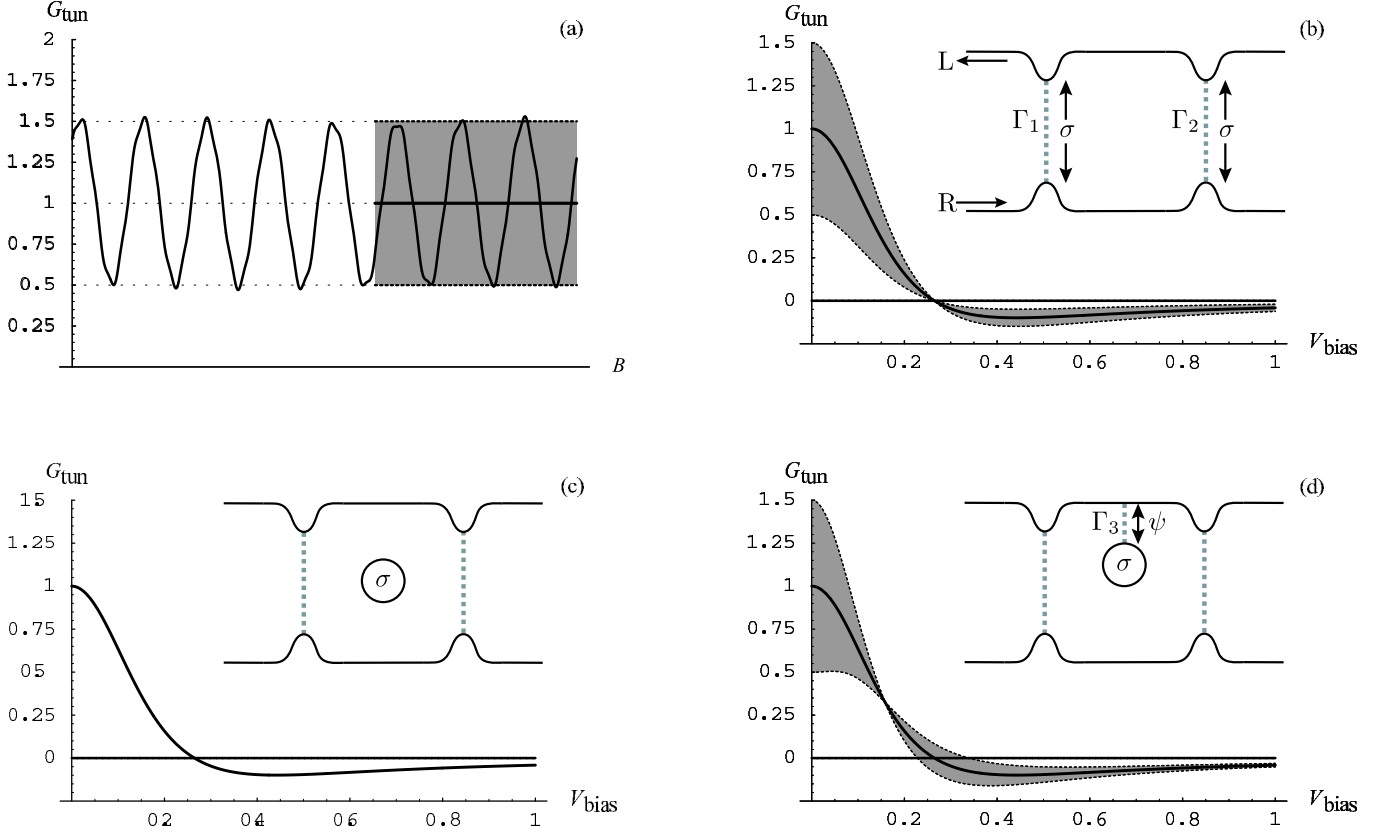


FIG. 1: Sketch of main predictions for interference behavior in tunneling conductance  $G_{\text{tun}}$ , for fixed temperature  $T$  and in the weak-tunneling regime. (a) At fixed bias voltage  $V_{\text{bias}}$  interference oscillations may be observed in  $G_{\text{tun}}$  as a function of magnetic field  $B$  (or any other Aharonov-Bohm phase changing variable), and *average* (thick line) and *oscillation amplitude* (grayed area) of the interference oscillations can be determined. (b) Behavior of average and amplitude of interference oscillations of  $G_{\text{tun}}$  as a function of  $V_{\text{bias}}$  for interferometer with no island. Both average and amplitude are given by  $f_0(V_{\text{bias}})$ . (c) For setup with a  $\sigma$ -quasiparticle at the central island the interference in  $G_{\text{tun}}$  is completely deconstructive and fully vanishes, i.e., the amplitude of oscillations is zero and only the average remains. (d) Interference is restored when the central island with the  $\sigma$ -quasiparticle can have  $\psi$ - $\psi$ -tunneling to one of the edges. However, average and amplitude now scale differently with bias, average like  $f_0(V_{\text{bias}})$  and amplitude like  $f_1(V_{\text{bias}})$ . This difference in scaling can be used to distinguish situations (b) and (d) when experimentally some interference is observed; as a function of  $T$  (b) and (d) also scale differently (not shown).  $V_{\text{bias}}$  is given in units of  $2\pi T/q$  and units of  $G_{\text{tun}}$  are arbitrary, with average and amplitude of  $G_{\text{tun}}$  at zero bias set to 1 and 0.5 respectively.

Hall state at filling fraction  $\nu = \frac{5}{2}$  is generally believed to be the Moore-Read Pfaffian state [5]; this claim is supported by numerical simulations for closed systems [13, 14, 15], but has not yet been established experimentally.

The edge states of the Moore-Read state are described by a free chiral charged boson plus free chiral Majorana fermion theory [15]. The corresponding conformal field theory is a central charge  $c = 1$  plus a  $c = \frac{1}{2}$  theory. Each edge in the quantum Hall liquid carries a sector label. Each sector forms an irreducible representation of the algebra of the electron operators. In CFT language electron operators are certain primary fields of the  $c = 1 + \frac{1}{2}$  theory. The different sectors are labeled by more general primary fields of the  $c = 1 + \frac{1}{2}$  theory. The non-abelian part of the Moore-Read state comes from the

$c = \frac{1}{2}$  Ising contribution which carries a primary field  $\sigma$  with fusion rules  $\sigma \times \sigma = \mathbb{1} + \psi$ ,  $\psi \times \psi = \mathbb{1}$ ,  $\sigma \times \psi = \sigma$ . The  $c = 1$  part is described by a bosonic field  $\phi$ .

The allowed sectors for each edge are constrained only by the condition that the fusion product of *all* edges in the quantum Hall liquid combined is in the identity representation. Otherwise we will consider the edges to be fully independent of each other, such that we let operators on different edges commute. We will consider the central island to be an edge as well, but with a finite edge length described by a discrete set of (no longer gapless) modes; a bulk quasiparticle can be considered as a zero-length island-edge that is labeled by a sector only but has no additional modes.

The Hamiltonian we consider is a copy of the free edge theory for each edge (see [15]) plus tunneling operators

which destroy a quasiparticle on one edge and create it on the edge on the opposite side of a point contact. As usual we restrict ourselves here to tunneling of the most relevant operator with the smallest fractional charge, the charge  $q = \frac{1}{4}$  quasiparticle with operator  $\sigma e^{\frac{1}{2}\phi}$  [15]. With  $\omega_J = qeV$  the applied bias voltage between edges L and R, and  $\Gamma_1$  and  $\Gamma_2$  the tunneling amplitudes at the two point contacts, the tunneling Hamiltonian becomes

$$H_{\text{tun}}(t) = \Gamma_1 e^{i\omega_J t} \sigma_L(t, x_1) e^{\frac{i}{2}\phi_L(t, x_1)} \sigma_R(t, x_1) e^{-\frac{i}{2}\phi_R(t, x_1)} \\ + \Gamma_2 e^{i\omega_J t} \sigma_L(t, x_2) e^{\frac{i}{2}\phi_L(t, x_2)} \sigma_R(t, x_2) e^{-\frac{i}{2}\phi_R(t, x_2)} \\ + \text{H.c.} \quad (1)$$

The tunneling current can be calculated in linear response to leading order in the tunneling amplitudes  $\Gamma_1$  and  $\Gamma_2$  by expressing the tunneling current in terms of a time-integral of time-ordered ground state correlation functions of CFT primary fields, which are known exactly both at zero temperature and finite temperature. We assume that the correlation function of a product of operators on different edges factors out completely into a product of correlation functions for each edge, and can be decoupled further into free boson and free Majorana fermions correlators. Following Ref. [16] we find for the tunneling current

$$I_{\text{tun}}(T, V) = |\Gamma_{\text{eff}}|^2 T^{-\frac{1}{2}} F_0\left(\frac{qV}{2\pi T}\right) \quad (2)$$

$$|\Gamma_{\text{eff}}|^2 = |\Gamma_1|^2 + |\Gamma_2|^2 + 2\text{Re}(\Gamma_1 \Gamma_2^* e^{i\theta_{AB}}) H(V, T, \bar{x}).$$

The generic structure of the tunneling current consists of three parts: a dimensionful powerlaw temperature factor, here  $T^{-1/2}$ , which can be read off directly from the CFT weights of the primary fields, a dimensionless function  $F_0(y)$  which gives the non-trivial dependence on bias voltage (as determined by the primary field weights [20]), and finally an effective tunneling amplitude  $|\Gamma_{\text{eff}}|^2$ . The effective tunneling amplitude is of the form typical for interference: a contribution from each tunneling path separately,  $|\Gamma_1|^2$  and  $|\Gamma_2|^2$ , plus a pure interference term which includes the Aharonov-Bohm phase  $\theta_{AB}$ .

The zero temperature limit,  $T \ll V$ , is properly taken care of as  $F_0(V/T) \sim (V/T)^{-1/2}$  in this limit. The interference-term also includes a dimensionless function  $H(V, T, \bar{x})$  which incorporates the effect of the distance  $\bar{x}$  between the two point contacts. Because the typical experimental  $\bar{x}$  is much smaller than  $V$  and  $T$  (when expressed in unit of length) and in this limit,  $H(V, T, \bar{x} = 0) = 1$ , we assume we can ignore  $H(T, V, \bar{x})$  altogether for our purposes.

Experimentally, it is not the tunneling current itself which is measured in FQH interferometers, but rather the differential tunneling conductance  $G_{\text{tun}} = \partial I_{\text{tun}} / \partial V$  [21], and we can write

$$G_{\text{tun}}(T, V) = |\Gamma_{\text{eff}}|^2 T^{-\frac{3}{2}} f_0\left(\frac{qV}{2\pi T}\right), \quad (3)$$

where  $f_0(y) = F'_0(y)$  and we absorbed the constant factors  $q/2\pi$  into the tunneling amplitudes  $\Gamma_i$ .

The differential tunneling conductance has a form

$$G_{\text{tun}}(T, V) = G_{\text{tun}}^{\text{ave}}(T, V) + G_{\text{tun}}^{\text{osc}}(T, V) \cos(\theta_{AB} + \theta_0), \\ G_{\text{tun}}^{\text{ave}}(T, V) = (|\Gamma_1|^2 + |\Gamma_2|^2) T^{-\frac{3}{2}} f_0\left(\frac{qV}{2\pi T}\right), \quad (4) \\ G_{\text{tun}}^{\text{osc}}(T, V) = 2|\Gamma_1 \Gamma_2| T^{-\frac{3}{2}} f_0\left(\frac{qV}{2\pi T}\right).$$

$G_{\text{tun}}^{\text{osc}}(T, V)$  describes the amplitude of the interference oscillation, and  $G_{\text{tun}}^{\text{ave}}(T, V)$  the average differential tunneling conductance. We note that both  $G_{\text{tun}}^{\text{osc}}(T, V)$  and  $G_{\text{tun}}^{\text{ave}}(T, V)$  scale as  $T^{-3/2}$ . In fact, they both depend on  $V/T$  in the same way.

### Vanishing interference with $\sigma$ quasiparticle on central island

The non-abelian statistics of the quasiparticles can be probed if the interferometer contains a non-trivial edge, e.g., a small central island (or bulk quasiparticle) in the  $\sigma$  sector, as depicted in the inset of Fig. 1c [22]. Now a non-trivial braiding, associated with a quasiparticle going around the path that is enclosed by the interferometer, enters the interference.

In order to account for the braiding in the interference we have to pay close attention to the fusion channels of our operators, guided by the following two principles: 1) a tunneling event creates quasiparticles on two edges which are in the *identity* channels as tunneling cannot change the sector of the total system, and 2) only operators which are in the identity channel on *each* edge have a non-vanishing expectation value. For abelian states one already used these principles as they encompass charge conservation. What non-abelian statistics adds to this is that for a given operator there can be more than one fusion channel and braiding can affect the channel.

Notationwise it is difficult to capture fusion and braiding concisely due to the inherent two-dimensional aspect of braiding; our notation is clearly not ideal but should suffice for our purposes.

We envision a product of operators which includes a tunneling operator of a  $\sigma$ -quasiparticle between left and right edges at location  $i$ , which we describe by  $[\sigma_L \sigma_R]_i^{\mathbb{1}}$ , where the superscript  $\mathbb{1}$  indicates that this tunneling operator is in the identity channel. Furthermore there is a similar tunneling operator at location  $j$ . The locations  $i$  and  $j$  and the Left and Right edges define an enclosed Island which can be endowed with a sector of the theory. If we let the sector of this island be the identity (as indicated by  $(\mathbb{1})_I$ , i.e., there is no non-trivial non-Abelian quasiparticle inside the enclosed island) we find the fusion of the two  $\sigma_L$  and two  $\sigma_R$  tunneling operators gives

rise to

$$[\sigma_L \sigma_R]_i^\dagger [\sigma_L \sigma_R]_j^\dagger \xrightarrow[\text{(\mathbb{1})}_I]{\text{fus.ch.}} \mathbb{1}_L \mathbb{1}_R \mathbb{1}_I + \psi_L \psi_R \mathbb{1}_I, \quad (5)$$

indicating that there exist two possible fusion channels including one that is the identity representation at all edges (left, right and island edges).

If the enclosed region contains a  $\sigma$  sector (i.e. a charge  $e/4$  non-Abelian quasiparticle), then the same tunneling operator  $[\sigma_L \sigma_R]_i^\dagger [\sigma_L \sigma_R]_j^\dagger$  will fuse differently

$$[\sigma_L \sigma_R]_i^\dagger [\sigma_L \sigma_R]_j^\dagger \xrightarrow[\text{(\sigma)}_I]{\text{fus.ch.}} \psi_L \mathbb{1}_R \psi_I + \mathbb{1}_L \psi_R \psi_I. \quad (6)$$

Note that if we move a charge  $e/4$  non-Abelian quasiparticle around another  $e/4$  non-Abelian quasiparticle, the two particles will each gain a neutral fermion [9, 10, 11, 12]. Such a non-trivial braiding is captured in Eq. (6) because the sector of the island has been altered. Since there is no channel in the identity representation at all three edges the expectation value of this operator is zero.

If one calculates the tunneling conductance for the case with a  $\sigma$ -quasiparticle present inside the interferometer one finds

$$G_{\text{tun}}(T, V) = G_{\text{tun}}^{\text{ave}}(T, V), \quad G_{\text{tun}}^{\text{osc}}(T, V) = 0. \quad (7)$$

The average tunneling conductance is still given by Eq. (4), but the interference term has vanished, see Fig. 1c. This follows from Eq. (6) because the operator that caused interference before now has a zero expectation value. An alternative explanation is that in encircling the island a  $\sigma$ -quasiparticle flips its internal two-dimensional state and ‘internal-spin-up’ and ‘internal-spin-down’ do not interfere. Also one can explain the vanishing interference as being able to tell which path the tunneling quasiparticle took, because this information can be determined from a hypothetical measurement of the sectors of the system before and after a tunneling event.

### INTERFERENCE RESTORED THROUGH $\psi$ - $\psi$ -TUNNELING

In the presence of a  $\sigma$  quasiparticle on the central island the interference vanished to leading order in  $\Gamma_1$  and  $\Gamma_2$ . Here we will consider the situation where the central island is close to one of the edges (which we choose to be the left edge). We will include a tunneling process between the central island and the left edge. For simplicity we restrict ourselves to the tunneling of the neutral fermions only, because this process leaves the sector of the island unaltered and furthermore this is a charge-neutral operation. We will see that the such higher order

tunneling processes can potentially restore some interference.

The tunneling of the neutral fermions is described by the tunneling Hamiltonian

$$H_{\text{tun}} \rightarrow H_{\text{tun}} + \Gamma_3 \psi_L(t, x_3) \psi_I(t), \quad (8)$$

where the tunneling amplitude  $\Gamma_3$  is real-valued to ensure hermiticity of the Hamiltonian (operators on different edges commute).

Inclusion of this operator has the potential to restore interference because the fusion channel of the operator

$$[\sigma_L \sigma_R]_i^\dagger [\psi_L \psi_I]^\dagger [\sigma_L \sigma_R]_j^\dagger \xrightarrow[\text{(\sigma)}_I]{\text{fus.ch.}} \mathbb{1}_L \mathbb{1}_R \mathbb{1}_I + \psi_L \psi_R \mathbb{1}_I \quad (9)$$

now has the full identity channel in it and hence a non-zero expectation value, whereas without the additional  $\psi_L \psi_I$  contribution it vanished.

We find a contribution to the tunneling conductance to leading order in  $\Gamma_3$  which contains some non-zero interference. The average value of the conductance is still unaltered to leading order, but the oscillation amplitude now behaves as

$$G_{\text{tun}}^{\text{osc}}(T, V) = 2|\Gamma_1 \Gamma_2 \Gamma_3| T^{-2} f_1\left(\frac{qV}{2\pi T}\right). \quad (10)$$

The interference now has a different power of  $T$ ,  $T^{-2}$  instead of  $T^{-3/2}$ , and a different dependence on  $V/T$  through a dimensionless function  $f_1(y)$ , as depicted in Fig. 1d.

Since the tunneling amplitudes  $\Gamma_i$  are generally considered to be unknown, except for that they should be small, the different scaling of the interference is the only way to tell one situation (interference with no central  $\sigma$ , Fig. 1b) from the other (interference with a central  $\sigma$  but also  $\psi$ - $\psi$ -tunneling, Fig. 1d).

An observation of  $\psi$ - $\psi$ -tunneling would indicate that the topologically protected zero-mode space is sensitive to ‘spin/qubit’ flips. It also help us to understand how the destruction of interference by non-Abelian statistics is restored as the non-Abelian particle between the two point contacts is moved near an edge.

### Calculation of the leading island tunneling contribution

The steady state tunneling current  $I_{\text{tun}}$  is calculated by expanding the time evolution operator, starting from an initial state  $|0\rangle$

$$I_{\text{tun}}(t) = \langle \varphi(t) | J | \varphi(t) \rangle \\ |\varphi(t)\rangle = \mathcal{T} \{ e^{-i \int_{-\infty}^t dt' [H_0 + H_{\text{tun}}(t')]} \} |0\rangle, \quad (11)$$

where  $\mathcal{T}\{\dots\}$  indicates time-ordering. Up to second order in  $H_{\text{tun}}$  this becomes

$$I_{\text{tun}}(t) = -i \int_{-\infty}^t dt' \langle 0 | [J(t), H_{\text{tun}}(t')] | 0 \rangle \\ + \int_{-\infty}^t dt_1 \int_{-\infty}^t dt_2 \left( -\langle 0 | \mathcal{T}\{J(t) H_{\text{tun}}(t_1) H_{\text{tun}}(t_2)\} | 0 \rangle \right. \\ \left. - \langle 0 | \mathcal{T}\{J(t) H_{\text{tun}}(t_1) H_{\text{tun}}(t_2)\} | 0 \rangle^* \right. \\ \left. + \langle 0 | H_{\text{tun}}(t_1) J(t) H_{\text{tun}}(t_2) | 0 \rangle \right) + \dots \quad (12)$$

The tunneling current operator  $J$  is given by  $iqe$  times  $H_{\text{tun}}$  from Eq. (1) with appropriate minus signs in the Hermitean conjugated part.

One term that appears in Eq. (12) is the correlation

$$\langle 0 | \mathcal{T}\{\sigma_L(t_1) \sigma_L(t_2) \sigma_R(t_1) \sigma_R(t_2) \psi_L(t_3) \psi_I(t_3)\} | 0 \rangle \quad (13)$$

From Eq. (6), we see that  $\sigma_L \sigma_L \sigma_R \sigma_R$  can fuse into  $\psi_L \psi_I$  if there is an  $e/4$  non-Abelian quasiparticle between the two junctions. Thus the above correlation is non-zero.

According to the conformal field theory, the time-ordered three-point-function (at zero temperature) is given by

$$\langle 0 | \mathcal{T}\{\sigma(t_1) \sigma(t_2) \psi(t_3)\} | 0 \rangle \\ = \frac{(\delta + i|t_1 - t_2|)^{3/8}}{(\delta + i|t_1 - t_3|)^{1/2} (\delta + i|t_2 - t_3|)^{1/2}}. \quad (14)$$

Using  $\langle 0 | \mathcal{T}\{\sigma_R(t_1) \sigma_R(t_2)\} | 0 \rangle \sim 1/(\delta + i|t_1 - t_2|)^{1/8}$ , we find

$$\langle 0 | \mathcal{T}\{\sigma_L(t_1) \sigma_L(t_2) \sigma_R(t_1) \sigma_R(t_2) \psi_L(t_3) \psi_I(t_3)\} | 0 \rangle \\ = \frac{(\delta + i|t_1 - t_2|)^{1/4}}{(\delta + i|t_1 - t_3|)^{1/2} (\delta + i|t_2 - t_3|)^{1/2}} \quad (15)$$

Here we have assumed that the correlation of  $\psi_I(t_3)$  with other operator does not depend on  $t_3$ .

The leading order contribution in the tunneling current is the familiar linear response result. But note that the next contribution contains both time-ordered and non-time-ordered parts. The non-time-ordered part is basically a Keldysh contour-ordered term; to compute its expectation value we have to analytically continue the time-ordered correlation functions.

We would like to have an expression for any ordering of the times  $t_1$ ,  $t_2$  and  $t_3$ , which we obtain by removing the absolute value bars,

$$\langle 0 | \sigma_L(t_1) \sigma_L(t_2) \sigma_R(t_1) \sigma_R(t_2) \psi_L(t_3) \psi_I(t_3) | 0 \rangle \\ = \frac{(\delta + i(t_1 - t_2))^{1/4}}{(\delta + i(t_1 - t_3))^{1/2} (\delta + i(t_2 - t_3))^{1/2}} \quad (16)$$

In these correlators  $\delta$  is the short-distance cutoff and we set the three-point-function prefactor to one.

In each term in the full expansion of Eq. (12) we assume we can decompose the expectation value of the product of operators into correlators of the  $c = 1$  and  $c = \frac{1}{2}$  theories for each edge separately, which takes the typical form

$$\langle (\sigma \sigma \psi)_L (\sigma \sigma)_R \psi_I \rangle \langle e^{\frac{i}{2}\phi} e^{-\frac{i}{2}\phi} \rangle_L \langle e^{\frac{i}{2}\phi} e^{-\frac{i}{2}\phi} \rangle_R. \quad (17)$$

The total scaling dimension of the above operators is 1. From the scaling consideration in the  $\int dt_1 \int dt_2$  integral, we find that such an operator will contribute  $\delta I_{\text{tun}} \propto T^{-1}$ , or more precisely

$$\delta I_{\text{tun}} = \frac{4qe}{\pi T} 2\text{Re}(\Gamma_1 \Gamma_2^* e^{i\theta_{AB}}) \Gamma_3 F_1\left(\frac{qV}{2\pi T}\right). \quad (18)$$

The calculation simplifies considerably because of a cancellation: the numerator in Eq. (16) gets cancelled by the product of the  $\phi$  two-point functions. Correlators on the non-dispersive island edge were set equal to a constant. Working out the double-integrals in Eq. (12) it turns out that the two time-ordered contributions depend on the cutoff and only the non-time-ordered part contributes in the limit of zero  $\delta$ . The contribution we find to first order in  $\Gamma_3$  in the tunneling current, working at finite temperature by letting  $(\delta + it)^g \rightarrow (\sin[\pi T(\delta + it)]/\pi T)^{-g}$  and setting  $x_i = 0$ , is Eq. (18) with  $F_1$  given by

$$F_1(y) = \int_0^\infty du_1 \sin(2yu_1) \int_0^{u_1} du_2 \frac{1}{\sqrt{\sinh u_2 \sinh(u_1 - u_2)}}. \quad (19)$$

We find that we can approximate the integral over  $u_2$  reasonably well with the function  $\pi/\cosh \frac{u_1}{2\sqrt{2}}$ . By taking the derivative with respect to  $V$ ,  $f_1(y) = F_1'(y)$ , and absorbing constants into the  $\Gamma_i$  we arrive at the expression Eq. (10) for the oscillation amplitude of the tunneling conductance. Note that the  $\Gamma_3$  contribution is pure interference *only*, there is no contribution to the average conductance.

### Flow to fixed point

Since the  $\Gamma_3$  contribution to the interference oscillation amplitude increases more rapidly than the average conductance as  $T$  goes to zero ( $T^{-2}$  versus  $T^{-3/2}$ ), we expect the present result to be unstable towards lowering of the temperature. And we can ask the question what fixed point this setup flows to; more carefully worded: by lowering the temperature and at the same time making  $\Gamma_1$  and  $\Gamma_2$  smaller as well such that tunneling between left and right edges is still weak, what interference pattern does this flow to? Since in our setup the present  $e/4$  non-Abelian quasiparticle on the island is not affected by the  $\psi$ - $\psi$  tunneling, the junction may flow to a non-trivial fixed point.

## SUMMARY

We calculate the scaling behavior of the non-linear I-V tunneling curves for a two-point-contact tunneling junction between two edges of the  $\nu = \frac{5}{2}$  non-abelian fraction quantum Hall states. Using the fusion rule of the tunneling operators, we can calculate the non-linear I-V tunneling curves for both cases with and without a  $e/4$  non-abelian quasiparticle between the two contacts. It was suggested that the presence of the  $e/4$  quasiparticle between the two contacts destroys the interference between the two tunneling paths. We show how to obtain such a result within a quantitative dynamical edge theory. Such a dynamical understanding allows us to calculate how the interference reappears as the  $e/4$  quasiparticle is moved closer to an edge. In particular, we consider the effect of a  $\psi$ - $\psi$  tunneling between the island (which traps a  $e/4$  quasiparticle) and an edge. We find that such a coupling between the island and the edge makes the interference pattern reappear. The scaling behavior of the induced interference pattern is calculated as well.

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- [20] Explicit form for  $F_0(y)$  can be read off from Eq. (A25) in Ref. [16] with  $g = \frac{1}{4}$ .
- [21] Typically, it is the the differential longitudinal resistance as function of bias current which is measured experimentally using lock-in techniques, but at a quantum Hall plateau this is equivalent to differential tunneling conductance as function of bias voltage.
- [22] Quasiparticles in the  $\sigma$  sector can only exist in pairs, so we assume that there is another  $\sigma$ -quasiparticle on another island in the FQH fluid outside the interferometer.